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AN APPROXIMATE ANALYSIS OF UNSTEADY VAPORIZATION NEAR THE STAGNATION POINT OF BLUNT BODIES

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SUMMARY

An approximate analysis of the unsteady vaporization of material from blunt bodies due to aerodynamic heating is presented. Two simultaneous nonlinear equations that can be solved to give the unsteady mass loss and the unsteady accumulation of heat by the remaining solid are derived. The vaporization of material by a steady stream (the conditions met with in laboratory experiments) is treated in detail.

INTRODUCTION

The cooling of blunt-nosed bodies in high-speed flight by ablation has been shown to be highly efficient when appropriate materials are used. A comprehensive review of cooling by mass addition to the boundary layer is included in reference 1. The use of materials which vaporize directly from the solid state appears to be particularly suitable in view of the fact that the mass lost from the body convects a large amount of heat away from the near vicinity of the stagnation point, as was shown by the analysis of reference 2. An important feature of any practical application of shielding by vaporization, however, is the unsteady accumulation of heat by the remaining solid material and the effect of this unsteadiness on the rate of vaporization. Any analysis of ablation-shield requirements for a reentry vehicle, for example, must consider the weight of material required to absorb that heat which is conducted toward the interior from the ablation surface. In the testing of materials, also, the conduction of heat may affect the experimental evaluation of the effective heat capacity of the material.

Previous work on the conduction problem (refs. 3 and 4) has neglected entirely the shielding effect of the gas layer and the problem has been solved under the assumption that the heat-transfer rate to the heated surface is known; however, the unsteady shielding effect of a molten film has been treated (ref. 5). When the vaporization is caused by aerodynamic heating the heat-transfer rate is itself a function of the mass-loss rate

(because of the gas layer shielding effect) and neglect of this effect can result in gross errors.

As was done in references 3 and 4, the material is considered to be of thickness greater than the heat penetration thickness so that the material can be considered semi-infinite in thickness for the purpose of analysis. The heat conduction problem before and during ablation is treated in a manner somewhat similar to that employed in reference 4, but the present method includes the important boundary-layer shielding effect.

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SYMBOLS

A	function of t (eq. 6)
a,b	arbitrary constants
c	specific heat
$\bar{c}_p$	effective specific heat of vapor in gas boundary layer
D	dimensionless mass loss per unit area (eq. (51))
E_n,E_n'	constants (eqs. (23) and (24))
h	heat-transfer coefficient
H_{eff}	effective heat capacity (eq. (55))
k	thermal conductivity
L	latent heat of vaporization
m	mass loss per unit area
N_{Pr}	Prandtl number
N_{Sc}	Schmidt number
P	dimensionless time
Q	heat transfer per unit area
q	heat-transfer rate per unit area

T temperature

t time

V dimensionless rate of mass loss per unit area (eq. (27))

$\bar{w}$ effective concentration of vapor in gas boundary layer

x coordinate along surface

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y coordinate normal to surface

z transformed coordinate normal to surface

α fractional temperature rise of vapor in gas boundary layer

β function of P_a (eq. (45))

δ dimensionless integral thickness (eq. (27))

ϵ enthalpy ratio (eq. (27))

ρ density

τ dimensionless time (eq. (27))

θ integral thickness of heated layer (eq. (2))

Subscripts:

a vaporization condition

b initial, unheated condition

e external to aerodynamic boundary layer

s surface

0 no vaporization

1 vapor

2 air

ANALYSIS

For the purpose of the present analysis it is assumed that the ablation material is semi-infinite in extent (fig. 1); such an approximation is valid near the stagnation point of blunt-nosed bodies when the thickness of the thermal boundary layer within the material is small compared with the total thickness. It is also assumed that mean constant values of ρ_b, c_b, and k_b can be used and that the ablation material vaporizes at a constant temperature T_a. The exact relation between the rate of vaporization and the surface temperature T_s is dependent on the phase equilibrium of the solid with its vapor, and vaporization occurs over a wide range of T_s; in practice, however, the rate of vaporization is negligibly small except when the surface temperature lies within a limited range which includes the mean value T_a used herein. Before heating begins the material is at uniform temperature T_b. The effects of radiation from the surface and of combustion at the surface are not considered, although they may be included with only minor alteration since the surface temperature is considered to be constant during ablation.

The Approximate Equations

The system of coordinates is such that the origin is always at the stagnation point in the vaporizing surface. Before ablation starts x is measured along, and y normal to, the surface; during ablation, however, coordinates (x,z) which move with the receding surface are used; thus

$$z = y + \frac{m(t)}{\rho_b}$$

and $z = 0$ is the ablation surface.

First, an energy balance equation is written as follows:

$$\underbrace{Q(t)}_{\text{Total heat input at surface}} = \underbrace{\left[c_b\left(T_a - T_b\right) + L\right]m}_{\text{Heat absorbed by ablated material}} + \underbrace{\rho_b c_b \int_{-\infty}^{0}\left(T - T_b\right)dz}_{\text{Heat accumulated by remaining material}} \qquad (1)$$

Introducing a thickness θ defined by

$$\theta = \int_{-\infty}^{0} \frac{T - T_b}{T_s - T_b}\, dz \tag{2}$$

equation (1) may be written

$$Q(t) = \left[c_b\left(T_a - T_b\right) + L\right]m + \rho_b c_b\left(T_s - T_b\right)\theta \tag{3}$$

or, in differential form,

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$$q(t) = \left[c_b\left(T_a - T_b\right) + L\right]\frac{dm}{dt} + \rho_b c_b \frac{d}{dt}\left[\left(T_s - T_b\right)\theta\right] \tag{4}$$

Equation (3) is an exact energy-balance equation which replaces the heat-conduction equation.

An additional equation is found by considering the boundary condition at the ablation surface; thus

$$\underbrace{q(t)}_{\substack{\text{Heat-transfer}\\ \text{rate to surface}}} = \underbrace{L\frac{dm}{dt}}_{\substack{\text{Rate of heat}\\ \text{absorbed by}\\ \text{ablation}}} + \underbrace{k_b\left(\frac{\partial T}{\partial z}\right)_{z=0}}_{\substack{\text{Heat-transfer}\\ \text{rate to interior}}} \tag{5}$$

Since θ is the only thickness associated with the problem, equation (5) can be written

$$q(t) = L\frac{dm}{dt} + A(t)k_b\frac{T_s - T_b}{\theta} \tag{6}$$

where $A(t)$ is a dimensionless coefficient, the main variation of $\left(\frac{\partial T}{\partial z}\right)_{z=0}$ being contained in the factor $\frac{T_s - T_b}{\theta}$.

At this point it is useful to ascertain which quantities are unknown; firstly, m and θ are the basic unknowns; secondly, the heat-transfer rate $q(t)$ is not known in general since the heat-transfer rate to the surface during ablation is itself a function of the mass loss rate. Throughout this analysis the quasisteady relation between heat-transfer rate and mass-loss rate as developed in reference 6 and used in reference 2 will be used; that is,

$$q(t) = q_0(t) - \alpha \bar{c}_p (T_e - T_a) \frac{dm}{dt} \tag{7}$$

where

$q_0(t)$ heat-transfer rate experienced by a nonablating body at a surface temperature T_a considered a known quantity

α factor which indicates that fraction of the temperature difference $T_e - T_a$ through which the mass m is raised during convection in the gas layer

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$\bar{c}_p$ effective mean specific heat of this mass in gaseous form

Expressions for α and $\bar{c}_p$ were derived in reference 6 and are:

$$\alpha = 1 - \frac{1}{3}\left(N_{Pr,a}\right)^{-0.6} \tag{8}$$

and

$$\bar{c}_p = c_{p,1}\bar{w} + c_{p,2}(1 - \bar{w}) \tag{9}$$

where $\bar{w}$ as a function of $N_{Sc,a}$ is given in reference 6.

The unknown quantity $q(t)$ is now eliminated from both equations (4) and (6) by use of equation (7) to give

$$q_0(t) = \left[c_b(T_a - T_b) + L + \alpha\bar{c}_p(T_e - T_a)\right]\frac{dm}{dt} + \rho_b c_b \frac{d}{dt}\left[(T_s - T_b)\theta\right] \tag{10}$$

and

$$q_0(t) = \left[L + \alpha\bar{c}_p(T_e - T_a)\right]\frac{dm}{dt} + A(t)k_b \frac{(T_s - T_b)}{\theta} \tag{11}$$

In order to solve equations (10) and (11) the function $A(t)$ should be known; then, the equations could be solved for T_s and θ during periods when ablation does not occur $\left(\frac{dm}{dt} \equiv 0\right)$ or for θ and m when ablation does occur $\left(T_s \equiv T_a\right)$.

The method used herein assumes that $A(t)$ is a slowly varying function which may be replaced by its quasisteady value; this assumption appears to be reasonable, inasmuch as the main variation of the term $k_b\left(\frac{\partial T}{\partial z}\right)_{z=0}$ in equation (5) is contained in the factor $k_b \frac{T_s - T_b}{\theta}$; however, the quasisteady ablation problem is easily solved (see appendix A) and gives simply

$$A \equiv 1 \tag{12}$$

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so that equation (11) becomes

$$q_0(t) = \left[L + \alpha\bar{c}_p\left(T_e - T_a\right)\right]\frac{dm}{dt} + k_b \frac{T_s - T_b}{\theta} \tag{13}$$

Equations (10) and (13) are the basic approximate equations to be used in the analysis that follows. The quantities T_s and θ are unknown when $\frac{dm}{dt} = 0$; the quantities m and θ are unknown when $T_s = T_a$; all other quantities in equations (10) and (13) are assumed to be known constants except $q_0(t)$ and $T_e(t)$, which are assumed to be known functions of t.

The validity of the approximation $A(t) \equiv 1$ is investigated by comparing the results derived from equations (10) and (13) with available exact (numerical) solutions.

Preablation Heating

The first quantities of interest are the time required to heat the surface up to the ablation temperature and the total amount of heat absorbed by the material during this period. When $\frac{dm}{dt} = 0$ (before ablation), equations (10) and (13) reduce to

$$q_0(t) = \rho_b c_b \frac{d}{dt}\left[\left(T_s - T_b\right)\theta\right] \tag{14}$$

and

$$q_0(t) = k_b \frac{T_s - T_b}{\theta} \tag{15}$$

Equation (14), however, may be integrated to give

$$\int_0^t q_0(t)dt = Q_0(t) = \rho_b c_b(T_s - T_b)\theta \tag{16}$$

and θ may be eliminated from equations (15) and (16) to give

$$q_0(t)Q_0(t) = \rho_b c_b k_b(T_s - T_b)^2 \tag{17}$$

The surface temperature during the preablation period is thus

$$T_s = T_b + \left[\frac{q_0(t)Q_0(t)}{\rho_b c_b k_b}\right]^{1/2} \tag{18}$$

The time t_a at which ablation starts is found by solving

$$q_0(t_a)Q_0(t_a) = \rho_b c_b k_b(T_a - T_b)^2 \tag{19}$$

and the heat absorbed by the material at this time is

$$Q_0(t_a) = \frac{\rho_b c_b k_b(T_a - T_b)^2}{q_0(t_a)} \tag{20}$$

As a check on the accuracy of equation (18), heating rates of the form

$$q_0(t) = a\left(\frac{t}{b}\right)^n \quad (n = 0, 1, 2, \ldots) \tag{21}$$

are considered, where a and b are arbitrary constants. Integration of equation (21) gives

$$Q_0(t) = \frac{ab}{n + 1}\left(\frac{t}{b}\right)^{n+1} \tag{22}$$

and equation (18) then gives

$$T_s = T_b + E_n a\left(\frac{b}{\rho_b c_b k_b}\right)^{1/2}\left(\frac{t}{b}\right)^{n+1/2} \tag{23}$$

where

$$E_n = (n + 1)^{-1/2}$$

The exact expression for T_s is derived in appendix B for the same heating rates (eq. (21)) and the solution may be written in the form

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$$T_s = T_b + E_n' a\left(\frac{b}{\rho_b c_b k_b}\right)^{1/2}\left(\frac{t}{b}\right)^{n+1/2} \tag{24}$$

where

$$E_n' = \frac{2^{2n+1}(n!)^2}{\pi^{1/2}(2n + 1)!}$$

A comparison of equations (23) and (24) shows that the solutions differ only by a constant factor; thus,

$$\frac{(T_s - T_b)_{approx}}{(T_s - T_b)_{exact}} = \frac{E_n}{E_n'} = \left(\frac{\pi}{n + 1}\right)^{1/2} \frac{(2n + 1)!}{2^{2n+1}(n!)^2}$$

The time t_a at which ablation starts is found by solving equation (19) and is

$$\left(\frac{t_a}{b}\right)_{approx} = E_n^{-\frac{2}{2n+1}}\left[\frac{\rho_b c_b k_b (T_a - T_b)^2}{a^2 b}\right]^{\frac{1}{2n+1}}$$

whereas the exact value is

$$\left(\frac{t_a}{b}\right)_{exact} = \left(E_n'\right)^{-\frac{2}{2n+1}} \left[\frac{\rho_b c_b k_b \left(T_a - T_b\right)}{a^2 h}\right]^{\frac{1}{2n+1}}$$

Ratios of the values of t_a and Q_a obtained by the approximate formulas to values obtained by the exact formulas have been calculated for $n = 0$ to 4 and are shown in table I, which follows:

Table I

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n	$\dfrac{(t_a)_{approx}}{(t_a)_{exact}}$	$\dfrac{(Q_a)_{approx}}{(Q_a)_{exact}}$
0	1.27	1.27
1	1.042	1.086
2	1.017	1.051
3	1.009	1.036
4	1.006	1.028

It is seen from table I that the fractional errors in t_a and Q_a are extremely small for smooth heating rates $(n > 1)$. When the heating rate is discontinuous at $t = 0$ $(n = 0)$, however, the errors are 27 percent.

The aerodynamic heating rates experienced by bodies in flight would probably correspond to $n > 1$. It is concluded that the approximation $A \equiv 1$ is valid for smooth preablation heating.

Unsteady Ablation

During periods of ablation when $T_s = T_a$, equations (10) and (13) reduce to

$$q_0(t) = \left[c_b\left(T_a - T_b\right) + L + \alpha\bar{c}_p\left(T_e - T_a\right)\right]\frac{dm}{dt} + \rho_b c_b\left(T_a - T_b\right)\frac{d\theta}{dt} \qquad (25)$$

and

$$q_0(t) = \left[L + \alpha\bar{c}_p\left(T_e - T_a\right)\right]\frac{dm}{dt} + \frac{k_b\left(T_a - T_b\right)}{\theta} \tag{26}$$

When the term $\alpha\bar{c}_p\left(T_e - T_a\right)$ is neglected (neglect of this term is not justified for most problems where the heating is caused aerodynamically), equations (25) and (26) are similar to equations derived in reference 4 where a polynomial temperature profile was assumed.

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The dimensionless variables τ, δ, V, and ϵ are now defined as

$$\left.\begin{aligned}
\tau &= \frac{t}{t_a} - 1 \\
\delta(\tau) &= \frac{\theta}{\theta_a} \\
V &= \frac{c_b\left(T_a - T_b\right) + L + \alpha\bar{c}_p\left(T_e - T_a\right)}{q_0}\frac{dm}{dt} \\
\epsilon &= \frac{c_b\left(T_a - T_b\right)}{c_b\left(T_a - T_b\right) + L + \alpha\bar{c}_p\left(T_e - T_a\right)}
\end{aligned}\right\} \tag{27}$$

and inserted into equations (25) and (26) to give

$$1 = V + \frac{Q_a}{q_a t_a}\frac{q_a}{q_0}\delta' \tag{28}$$

$$1 = V(1 - \epsilon) + \frac{q_a}{q_0}\frac{1}{\delta} \tag{29}$$

where the relations

$$Q_a = \rho_b c_b\left(T_a - T_b\right)\theta_a$$

$$q_a = \frac{k_b\left(T_a - T_b\right)}{\theta_a}$$

and

$$\delta' = \frac{d\delta}{d\tau}$$

have been used in equations (28) and (29). In general, V, ϵ, δ, and $\frac{q_a}{q_0}$ are all functions of τ. Equations (28) and (29) form a system of two nonlinear differential equations from which V and δ can be determined as functions of τ; q_0 is a known function (being the heat-transfer rate to the nonablating body); and q_a, Q_a, θ_a, and t_a are determined from the solution of the preablation heating problem.

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As a further check on the approximation $A \equiv 1$, equations (28) and (29) are solved for the simple case

$$q_0(t) = q_a = \text{Const}$$

and

$$\alpha\bar{c}_p(T_e - T_a) = 0$$

which is the case considered in reference 3.

The time t_a when ablation starts is given by

$$\begin{aligned} Q_0(t_a)q_0(t_a) &= q_a{}^2 t_a \\ &= \rho_b c_b k_b (T_a - T_b)^2 \end{aligned} \tag{30}$$

Therefore,

$$t_a = \frac{\rho_b c_b k_b}{q_a{}^2}(T_a - T_b)^2 \tag{31}$$

The thickness θ_a at this time is given, from equation (15), as

$$\theta_a = k_b \frac{(T_a - T_b)}{q_a} \tag{32}$$

Equations (28) and (29) now reduce to

$$1 = V + \delta' \tag{33}$$

$$1 = V(1 - \epsilon) + \frac{1}{\delta} \tag{34}$$

Elimination of V from equations (33) and (34) gives a nonlinear differential equation for δ

$$\delta' = \frac{1}{1 - \epsilon}\left(\frac{1}{\delta} - \epsilon\right) \tag{35}$$

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which has the solution, for $\epsilon \neq 0$,

$$\tau = \frac{1 - \epsilon}{\epsilon}\left(1 - \delta - \frac{1}{\epsilon} \log \frac{1 - \epsilon\delta}{1 - \epsilon}\right) \tag{36}$$

and for $\epsilon = 0$,

$$\tau = \frac{1}{2}\left(\delta^2 - 1\right) \tag{37}$$

The dimensionless mass-loss rate V is given by

$$V = \frac{1}{1 - \epsilon}\left(1 - \frac{1}{\delta}\right) \tag{38}$$

where δ is known implicitly. As $\tau \to \infty$, steady ablation is approached and, as can be seen from equation (33), $V \to 1$, and from equation (35), $\delta \to \frac{1}{\epsilon}$.

A comparison of the results given by equations (36) or equations (37) and (38) with results of reference 3 (which correspond to $n = 0$) is shown in figure 2. (The enthalpy parameter ϵ herein is related to the enthalpy parameter m of reference 3 by $\epsilon = \dfrac{m}{m + \frac{1}{2}\pi^{1/2}}$, and the dimensionless mass-loss rate V is related to the quantity μ of reference 3 by $(1 - \epsilon)V = \mu$.)

The dimensionless quantity $V(\tau)$ is noted to be in good agreement with the results of reference 3, although the quantity t_a which relates t with τ $\left(\text{that is, } \tau = t/t_a\right)$ is in considerable error. (See table I with $n = 0$.) This agreement for the dimensionless parameters suggests

that the approximate method may be used with accuracy to find the dimensionless quantities V and δ in terms of τ but that the value of t_a obtained from the preablation heating problem (which involves only a linear heat-conduction equation) should be obtained by a more accurate analysis when discontinuities in the heating rate occur.

The use of the approximation $A \equiv 1$ is seen to agree exactly with the steady-state solution, gives good agreement with exact results for the preablation heating problem except when discontinuous heating rates are involved, and agrees with an available numerical solution of an unsteady ablation problem. This agreement is considered sufficient justification for the use of equations (10) and (13) for determining rate of vaporization and accumulation of heat during the entire heating period.

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APPLICATION TO TESTING OF ABLATION MATERIALS

The analysis is now applied to the type of ablation experienced by a blunt-nosed body placed in a steady stream, that is, in a stream having constant velocity and temperature at far distances upstream of the body. These are the conditions which prevail in the simplest type of experiment designed to investigate the manner of ablation of different materials.

The heat-transfer rate to the nonablating body is assumed to be of the form

$$q_0 = h\left(T_e - T_s\right)$$

where h is a constant heat-transfer coefficient but T_s varies $\left(T_b < T_s < T_a\right)$ during the preablation heating period.

Preablation Heating

The heating rate experienced by the model is discontinuous at $t = 0$; the exact solution to the conduction equation is therefore used. The relevant solution to the conduction equation is

$$\frac{T - T_b}{T_a - T_b} = \frac{T_e - T_b}{T_a - T_b}\left\{\operatorname{erfc}\frac{-y}{2\left(\frac{k_b t}{\rho_b c_b}\right)^{1/2}} - e^{\frac{h^2 t}{\rho_b c_b k_b} + \frac{h}{k_b} y}\operatorname{erfc}\left[\left(\frac{h^2 t}{\rho_b c_b k_b}\right)^{1/2} - \frac{y}{2\left(\frac{k_b t}{\rho_b c_b}\right)^{1/2}}\right]\right\} \tag{39}$$

As indicated previously, the ablation temperature T_a depends on the phase equilibrium of the solid with its vapor and in theory may be determined from the Clausius-Clapeyron relation; in practice, however, it is simpler to make pyrophotometric observations of the ablation surface to determine T_a.

The preablation heating period t_a is found by letting $y = 0$ and $T = T_a$; thus,

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$$\frac{T_a - T_b}{T_e - T_b} = 1 - e^{\frac{h^2 t_a}{\rho_b c_b k_b}} \operatorname{erfc} h\left(\frac{t_a}{\rho_b c_b k_b}\right)^{1/2}$$

$$= 1 - e^{P_a} \operatorname{erfc}\left(P_a\right)^{1/2} \tag{40}$$

where

$$P_a = \frac{h^2 t_a}{\rho_b c_b k_b}$$

For large values of P_a

$$\frac{T_a - T_b}{T_e - T_b} \rightarrow 1 - \left(\pi P_a\right)^{-1/2}$$

or

$$\frac{T_e - T_a}{T_e - T_b} = \left(\pi P_a\right)^{-1/2} \tag{41}$$

In terms of t_a,

$$t_a \rightarrow \left(\frac{T_e - T_b}{T_e - T_a}\right)^2 \frac{\rho_b c_b k_b}{h^2 \pi} \tag{42}$$

for small values of $\frac{T_e - T_a}{T_e - T_b}$. Figure 3 shows the variation of $\frac{T_e - T_a}{T_e - T_b}$ with P_a.

The heat content of the material when $t = t_a$ is obtained, by using equation (39), as

$$Q_a = \rho_b c_b (T_a - T_b)\theta_a$$

$$= \int_0^{t_a} h(T_e - T_s)\,dt$$

$$= \frac{\rho_b c_b k_b}{h}(T_e - T_b) \int_0^{P_a} e^P \operatorname{erfc} P^{1/2} dP \qquad (43)$$

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Integration by parts of the right-hand side of equation (43) yields

$$(T_a - T_b)\theta_a = \frac{k_b}{h}(T_e - T_b)\left(\frac{2}{\pi^{1/2}} P_a{}^{1/2} + e^{P_a} \operatorname{erfc} P_a{}^{1/2} - 1\right)$$

or, when equation (40) is used,

$$\theta_a = \frac{k_b}{h}\left[\frac{2\left(\frac{P_a}{\pi}\right)^{1/2}}{1 - e^{P_a} \operatorname{erfc} P_a{}^{1/2}} - 1\right] \qquad (44)$$

Unsteady Ablation

During ablation, no exact method is available for the solution of the heating problem, and the approximate equations (28) and (29) are used. Now $q_a/q_0 = 1$ and the constant quantity $Q_a/q_a t_a$ is determined as

$$\frac{Q_a}{q_a t_a} = \frac{\frac{2}{\pi^{1/2}} P_a{}^{1/2} + e^{P_a} \operatorname{erfc} P_a{}^{1/2} - 1}{P_a e^{P_a} \operatorname{erfc} P_a{}^{1/2}}$$

$$= \frac{1}{\beta(P_a)} \qquad (45)$$

where

$$Q_a = \rho_b c_b (T_a - T_b)\theta_a$$

$$= \frac{\rho_b c_b k_b}{h}(T_a - T_b)\left(\frac{\frac{2}{\pi^{1/2}} P_a^{1/2}}{1 - e^{P_a}\operatorname{erfc} P_a^{1/2}} - 1\right)$$

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$$q_a = h(T_e - T_a)$$

$$= h(T_a - T_b)\left(\frac{T_e - T_b}{T_a - T_b} - 1\right)$$

$$= h(T_a - T_b)\,\frac{e^{P_a}\operatorname{erfc} P_a^{1/2}}{1 - e^{P_a}\operatorname{erfc} P_a^{1/2}}$$

and

$$t_a = \frac{\rho_b c_b k_b}{h^2} P_a$$

Equations (28) and (29) now reduce to

$$1 = V + \frac{1}{\beta(P_a)}\delta' \tag{46}$$

$$1 = V(1 - \epsilon) + \frac{1}{\delta} \tag{47}$$

which have the solution for $\epsilon \neq 0$,

$$\beta(P_a)\tau = \frac{1 - \epsilon}{\epsilon}\left(1 - \delta - \frac{1}{\epsilon}\log\frac{1 - \epsilon\delta}{1 - \epsilon}\right) \tag{48a}$$

or, for $\epsilon = 0$,

$$\beta(P_a)\tau = \frac{1}{2}(\delta^2 - 1) \tag{48b}$$

and

$$V = \frac{1}{1 - \epsilon}\left(1 - \frac{1}{\delta}\right) \tag{49}$$

The function $V(\tau,\epsilon)$ is shown in figure 4 as a function of $\beta(P_a)\tau$. It is seen that equations (48a) and (48b) differ from equations (36) and (37) only by a factor $\beta(P_a)$ which appears on the left-hand side.

The variation of $\beta(P_a)$ with P_a is shown in figure 3. For small values of P_a, use of the relations

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$$\operatorname{erfc} P_a^{1/2} = 1 - \frac{2}{\pi^{1/2}} P_a^{1/2} + \frac{2}{\pi^{1/2}} \frac{P_a^{3/2}}{3} + \text{Order}\left(P_a^{5/2}\right)$$

and

$$e^{P_a} = 1 + P_a + \text{Order}\left(P_a^{2}\right)$$

with equation (45) gives

$$\beta(P_a) = 1 - \frac{2}{3\pi^{1/2}} P_a^{1/2} + \text{Order}(P_a)$$

For large values of P_a, since $e^{P_a}\operatorname{erfc} P_a^{1/2} \rightarrow (\pi P_a)^{-1/2}$ as $P_a \rightarrow \infty$, equation (45) gives $\beta(P_a) \rightarrow 1/2$; thus

$$1 \geqq \beta(P_a) > 1/2$$

The total dimensionless mass loss D is found by integration of equation (46) and is written

$$\beta(P_a)D = \beta(P_a)\tau + 1 - \delta \tag{50}$$

where

$$D = \frac{m}{q_a t_a}\left[c_b(T_a - T_b) + L + \alpha\bar{c}_p(T_e - T_a)\right] \tag{51}$$

Figure 5 shows $\beta(P_a)D$ as a function of $\beta(P_a)\tau$.

Steady Ablation

The steady state is approached when $\delta' \to 0$; then $V \to 1$ and

$$\frac{dm}{dt} \to \frac{q_a}{c_b(T_a - T_b) + L + \alpha\bar{c}_p(T_e - T_a)} = \dot{m} \tag{52}$$

where

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$$q_a = h(T_e - T_a)$$

The temperature distribution is that given in appendix A

$$\frac{T - T_b}{T_a - T_b} = e^{\frac{\dot{m}c_b}{k_b}z}$$

$$= e^{\frac{h(T_e - T_a)\frac{c_b}{k_b}z}{c_b(T_a - T_b) + L + \alpha\bar{c}_p(T_e - T_a)}} \tag{53}$$

It is seen that for large values of $T_e - T_a$ the temperature distribution takes the limiting form

$$\frac{T - T_b}{T_a - T_b} = e^{\frac{h}{\alpha k_b}\frac{c_b}{\bar{c}_p}z} \tag{54}$$

DISCUSSION

The preceding analysis of ablation due to aerodynamic heating by a steady flow of hot air may be used in conjunction with experimentally determined values of the heat-transfer rate and mass loss to determine the suitability of materials for use as an ablation shield.

The behavior of ablation material during preablation heating is of extreme interest when the heating rate is small and the time t_a may be relatively large; under these conditions the material may become soft and change shape in an undesirable manner.

The first quantity of interest is the ablation temperature T_a; this value may be determined from the analysis of preablation heating. When h and $\rho_b c_b k_b$ are known, equation (40) may be used to find $\frac{T_a - T_b}{T_e - T_b}$ using observed values of t_a, so that T_a is then known in terms of T_e and T_b.

The quantity of primary interest during ablation is the effective heat capacity of the material which is conveniently defined for steady-state ablation as

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$$H_{eff} = \frac{q_a}{\dot{m}} = c_b(T_a - T_b) + L + \alpha\bar{c}_p(T_e - T_a) \tag{55}$$

which is obtained from equation (10) with $T_s = T_a$ and θ = constant. The quantity H_{eff} can be determined from equation (51) as

$$H_{eff} = \frac{Dq_a t_a}{m}$$

Thus, H_{eff} may be calculated by use of observed values of the total mass loss m, the time of the experiment t, and the associated value of D found from figure 5 as a function of $\tau = \frac{t}{t_a} - 1$ and $\beta(P_a) = \beta\left(\frac{h^2 t_a}{\rho_b c_b k_b}\right)$, which is shown as a function of P_a in figure 3. It is seen that in this way the steady-state value of H_{eff} can be determined, although the steady state is not necessarily achieved experimentally.

If a series of tests is carried out, in which the external temperature (i.e., the stagnation temperature near the stagnation point) takes various constant values, experimental values of $c_b(T_a - T_b) + L$ and $\alpha\bar{c}_p$ can be determined from H_{eff} through equation (55).

CONCLUSION

An approximate analysis has been presented which describes the rate of mass loss from, and the accumulation of heat by, material which is vaporizing because of aerodynamic heating. In this analysis the

combined effects of boundary-layer shielding and heat conduction to the interior are represented and the approximation involved is justified by comparing the results with known exact calculations.

The analysis is then applied to the vaporization of blunt-nosed bodies placed in a steady stream - for example, the conditions met with in a simple experiment. It is shown that the steady value of the effective heat capacity of the material can be determined even when the heating period during the experiment may not be sufficiently long to establish the steady state.

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Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., June 12, 1959.

APPENDIX A

STEADY-STATE SOLUTION

The heat conduction equation for a steadily ablating solid is

$$k_b \frac{d^2T}{dz^2} = \dot{m}c_b \frac{dT}{dz} \tag{A1}$$

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Equation (A1) expresses a balance of diffusion of heat to the interior and convection of heat by the material (which is moving with velocity $\dot{m}/\rho_b$ toward its ablation surface.)

The solution is written

$$\frac{T - T_b}{T_a - T_b} = e^{\frac{\dot{m}c_b}{k_b}z} \tag{A2}$$

which is seen to satisfy the conditions: at $y = 0$, $T = T_a$; at $y \rightarrow -\infty$, $T = T_b$.

From equation (A2) the integral thickness θ is found as

$$\theta = \int_{-\infty}^{0} e^{\frac{\dot{m}c_b}{k_b}z} dz = \frac{k_b}{\dot{m}c_b} \tag{A3}$$

The general equations during ablation (eqs. (10) and (11) derived in the main text) become:

$$q_0 = \left[c_b\left(T_a - T_b\right) + L + \alpha\bar{c}_p\left(T_e - T_a\right)\right]\frac{dm}{dt} + \rho_b c_b\left(T_a - T_b\right)\frac{d\theta}{dt}$$

and

$$q_0 = \left[L + \alpha\bar{c}_p\left(T_e - T_a\right)\right]\frac{dm}{dt} + A(t)\,\frac{k_b\left(T_a - T_b\right)}{\theta}$$

from which the following expression is obtained when $\frac{d\theta}{dt} = 0$ (steady state)

$$\theta = \frac{Ak_b}{\dot{m}c_b} \tag{A4}$$

A comparison of equation (A3) with equation (A4) shows that for steady ablation

$$A \equiv 1$$

It is noted that this value of A is independent of q_0.

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APPENDIX B

EXACT PREABLATION HEATING SOLUTIONS

Before ablation starts the heating problem is represented by the following differential equation and boundary conditions:

$$\frac{\partial T}{\partial t} = \frac{k_b}{\rho_b c_b} \frac{\partial^2 T}{\partial y^2} \qquad (0 > y > -\infty) \qquad \text{(B1)}$$

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with

$$\left.\begin{aligned} q_0(t) &= k_b \frac{\partial T}{\partial y} && (y = 0, \quad 0 < t < t_a) \\ T &\rightarrow T_b && (y \rightarrow -\infty, \quad 0 < t < t_a) \end{aligned}\right\} \quad \text{(B2)}$$

$$T = T_b \qquad (0 > y \rightarrow -\infty, \quad t = 0) \qquad \text{(B3)}$$

Here it is assumed that $q_0(t)$ is a monotonic increasing function of t in the range $0 < t < t_a$; for example,

$$q_0 = a\left(\frac{t}{b}\right)^n \qquad (n > 0) \qquad \text{(B4)}$$

The solution of equations (B1) to (B4) is then found by straightforward application of Laplace transform methods and is written

$$T - T_b = 2^{n+1} a\left(\frac{b}{\rho_b c_b k_b}\right)^{\frac{1}{2}}\left(\frac{t}{b}\right)^{n+\frac{1}{2}} i^{2n+1}\text{erfc} - \frac{y}{2}\left(\frac{\rho_b \bar{c}_b}{k_b t}\right)^{\frac{1}{2}} \qquad \text{(B5)}$$

where

$$i^m\text{erfc } w = \int_w^\infty i^{m-1}\text{erfc } \xi \, d\xi$$

and is the mth integral of the complementary error function. (See ref. 7.)

The surface temperature is found by putting $y = 0$; that is,

$$T_s - T_b = 2^{2n+1} a \left(\frac{b}{\rho_b c_b k_b}\right)^{\frac{1}{2}} \left(\frac{t}{b}\right)^{n+\frac{1}{2}} i^{2n+1}\operatorname{erfc} 0 \tag{B6}$$

where

$$i^{2n+1}\operatorname{erfc} 0 = \frac{n!}{2\left(\frac{1}{2}!\right)(2n + 1)!} = \frac{n!}{\pi^{\frac{1}{2}}(2n + 1)!}$$

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Thus the surface temperature T_s increases with t according to the relation

$$T_s - T_b = E_n' a \left(\frac{b}{\rho_b c_b k_b}\right)^{\frac{1}{2}} \left(\frac{t}{b}\right)^{n+\frac{1}{2}} \tag{B7}$$

where

$$E_n' = \frac{2^{2n+1}(n!)^2}{\pi^{\frac{1}{2}}(2n + 1)!} \tag{B8}$$

The coefficient E_n' is compared with that given by the approximate analysis in the main text.

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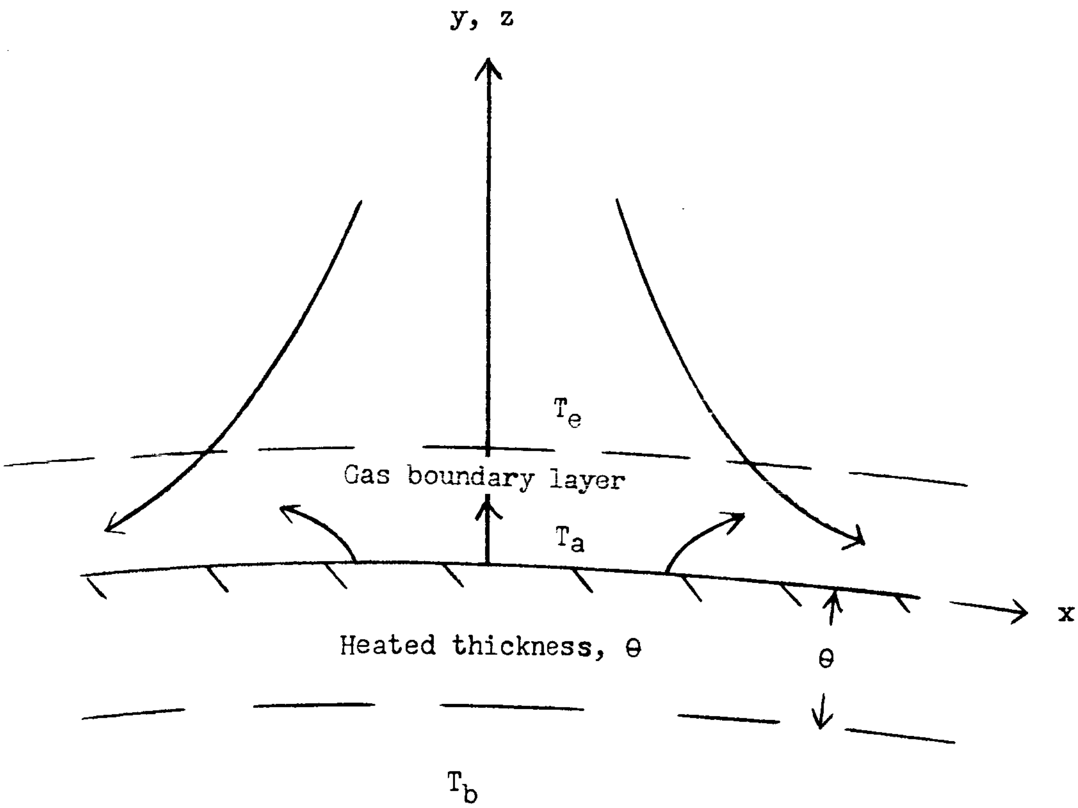

Figure 1.- Geometry near stagnation point.

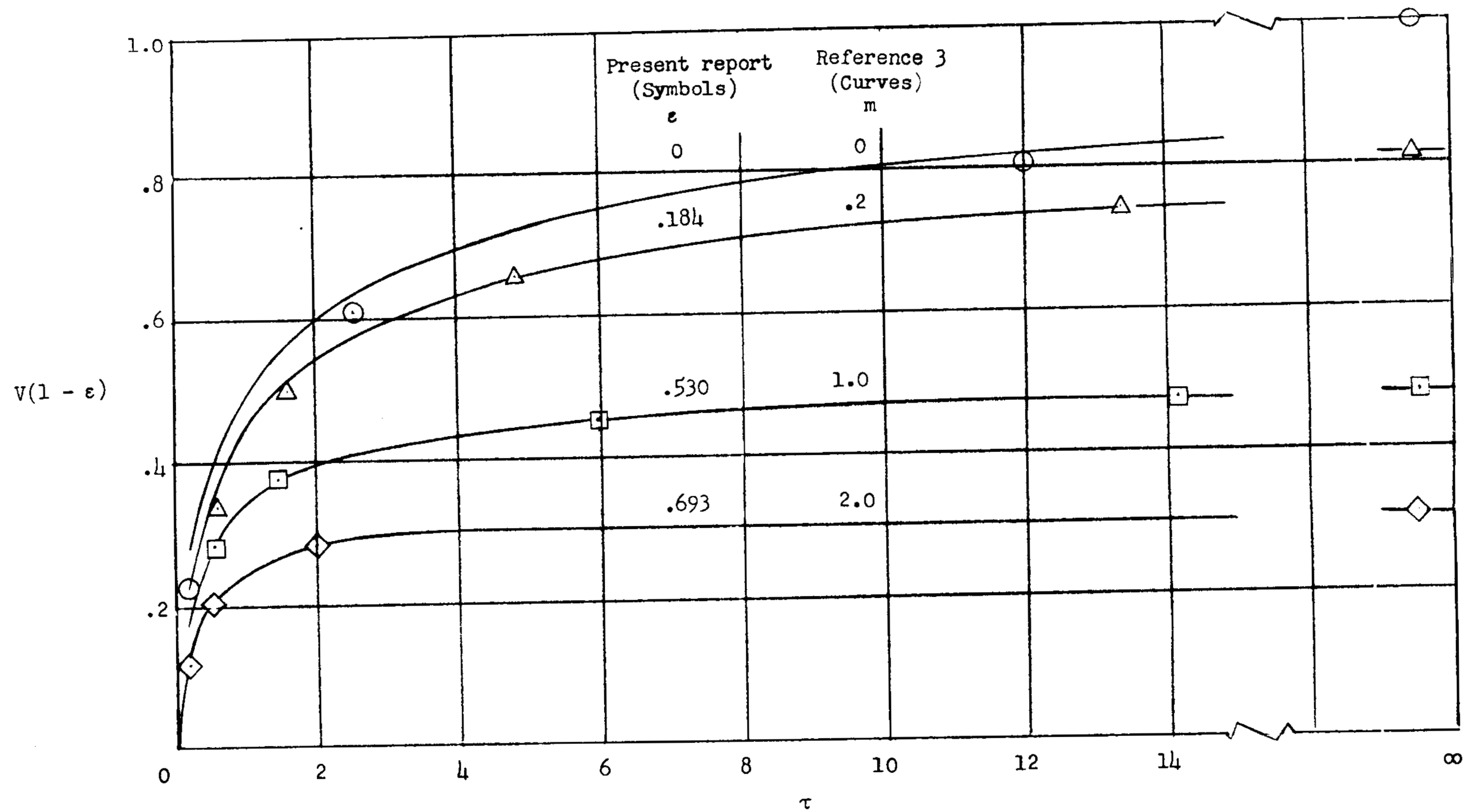

Figure 2.- Comparison of present theory with reference 3.

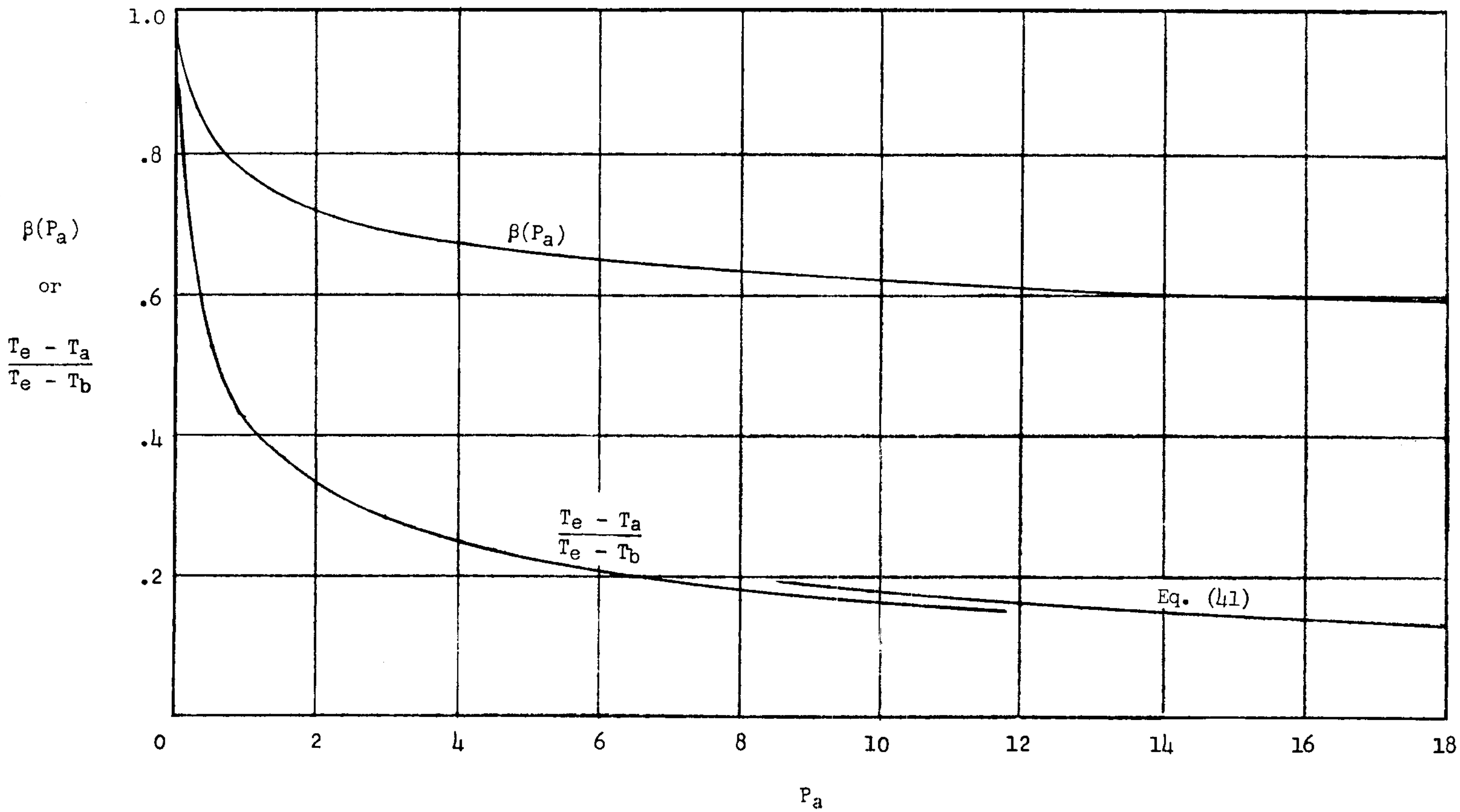

Figure 3.- Functions of the parameter β.

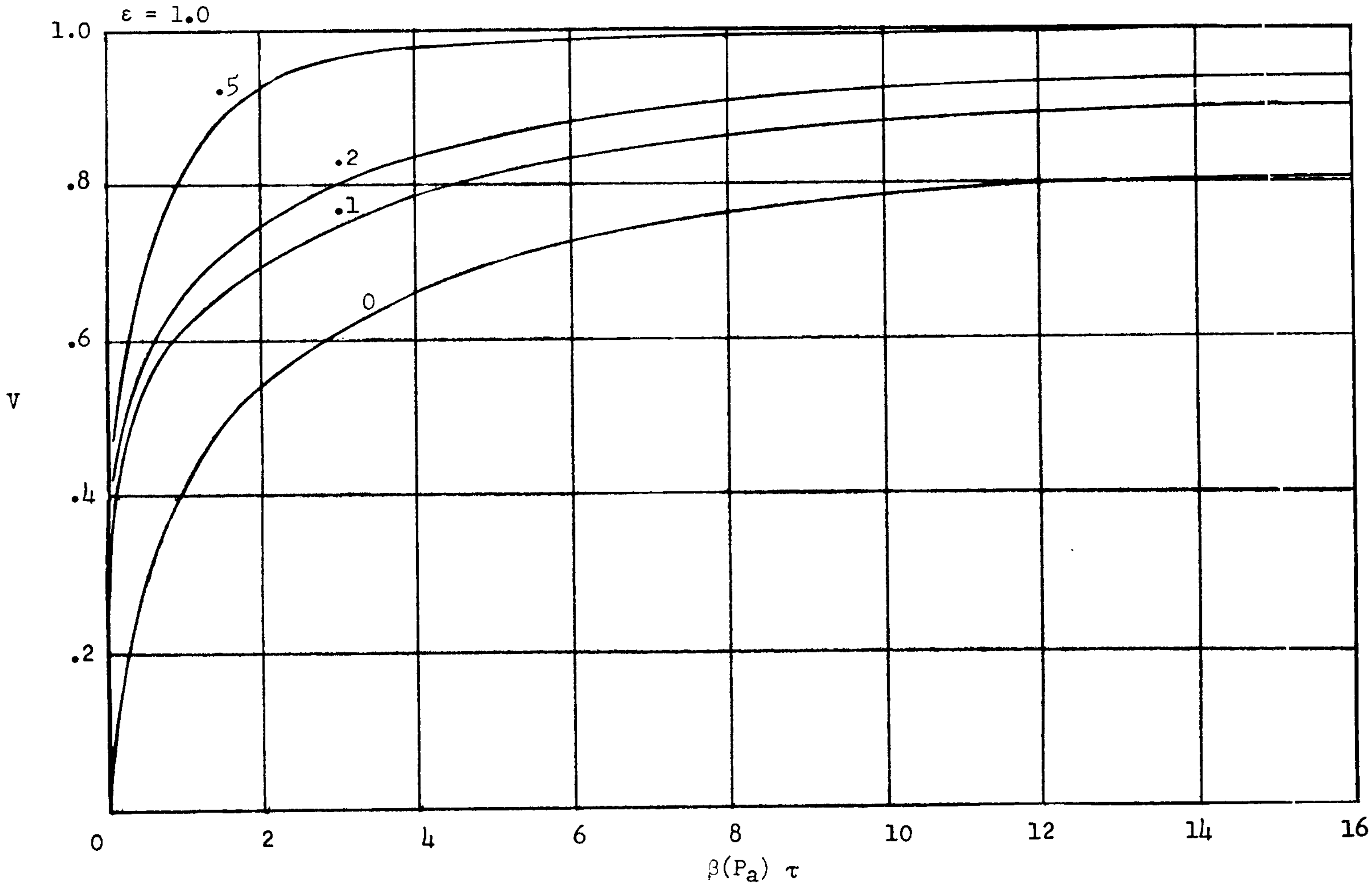

Figure 4.- Variation of dimensionless mass-loss rate V with dimensionless time parameter $\beta(P_a)\tau$.

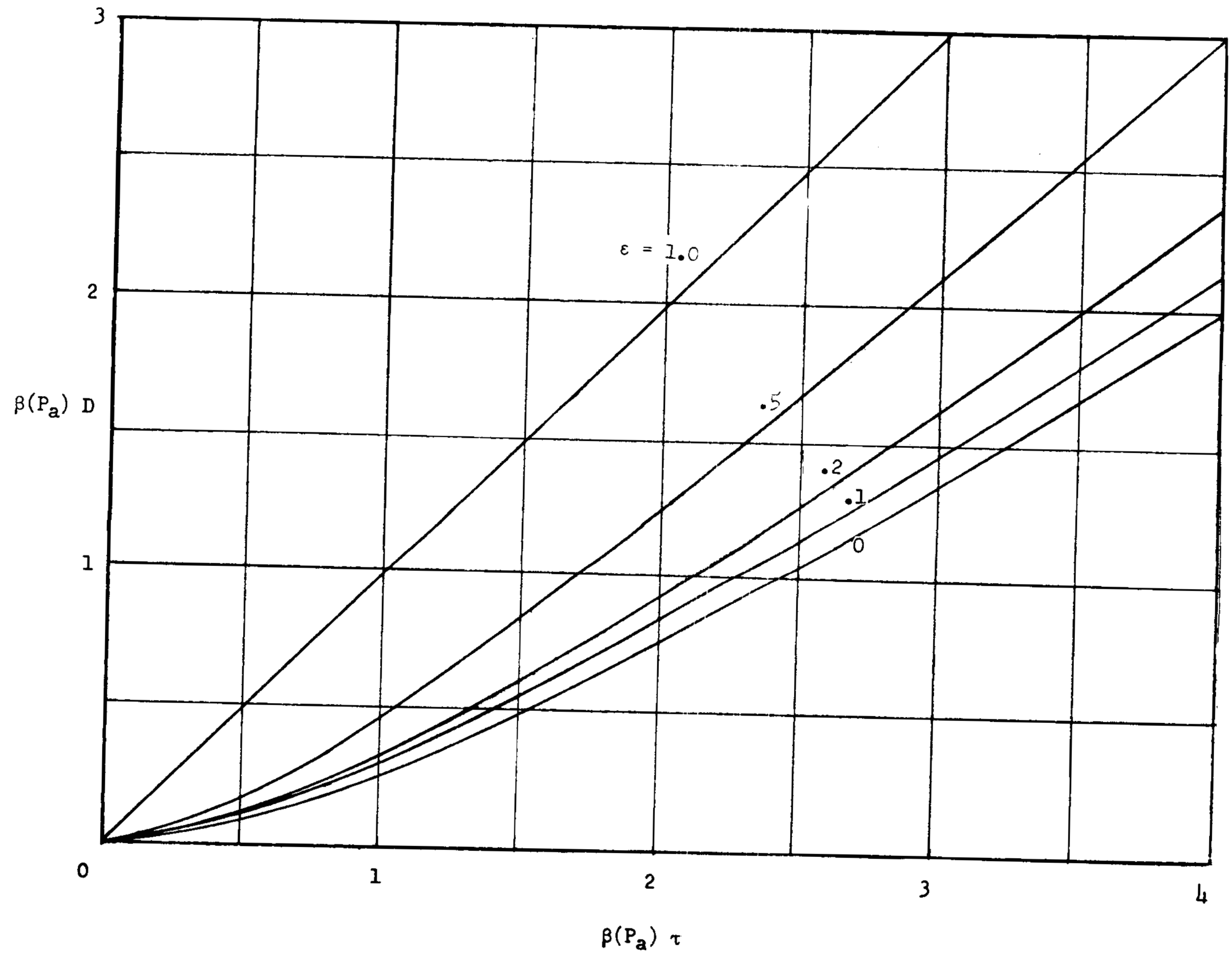

Figure 5.- Variation of dimensionless mass-loss parameter βD with dimensionless time parameter $\beta(P_a)\tau$.

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